Reactive Power Compensation in Mechanical Systems

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ABSTRACT

In this paper the problem of energy consumption in mechanical systems is approached from an electrical engineering point of view. To achieve this objective classical concepts in electrical networks theory like apparent power, reactive power and power factor have been extended to mechanical systems. This paper focus on the role of springs in mechanical systems to avoid power oscillations between joint actuators and loads. Such oscillations are a major problem because they unnecessarily increases the mean-square value of joint torques and by consequence Joule effect losses in the actuators. The minimization of these oscillations is known as "reactive power compensation". The main points illustrated in this paper are the fundamental limitations on reactive power compensation and the negative effect on the energy consumption of the harmonic content of the reference trajectory.

1 Introduction

In rotational mechanical systems like robot joints, the instantaneous power delivered by a motion actuator is given by the product between the joint torque and the joint velocity. If the mechanical load introduces a phase shift between these two variables, the sign of the instantaneous power is not constant. As a consequence the flow of energy between the actuator and the load is bidirectional. For a passive load, it implies that a part of the received energy is stored and subsequently forwarded to the actuator. This phenomenon entails two main problems. The first one is that most actuators do not have energy recovery capabilities so this forwarded energy is lost by Joule effect. The second one is that the mean-square value of the torque required to produce a given motion is unnecessarily incremented because of the additional transfer of energy from the actuator to the load.

In the same way as in electrical networks capacitors are used to compensate phase shifts between voltage and current created by inductive loads, we show that springs play the same role in mechanical systems. Thus, in both electrical and mechanical systems, phase shift compensation between inputs and outputs is a fundamental issue for the improvement of the energy transfer between a source and a load. Despite of this similarity, efficient power transmission in mechanical systems is far from being evident. The main difficulties arise from the non-sinusoidal nature of joint robotic motions and the non linear dynamics present in most mechanical systems.

It is important to note that in the case of nonlinear systems, phase shift compensation between torque and velocity does not necessarily guarantee an unidirectional flow of energy. Moreover, only in the case of linear systems excited with sinusoidal inputs, phase-shift compensation guarantees an efficient energy transfer between source and load [2].

The objective of this paper is to show the applicability of recent theoretical advances [3], [5], [6] in power-factor compensation of nonlinear electrical networks excited with non-sinusoidal signals for the minimization of energy consumption in mechanical systems. To achieve this objective it has been necessary to generalize classical concepts in electrical engineering like power factor, apparent power and reactive power.

This paper is organized as follows. In section 2 the problem statement and the assumptions for the rest of the paper are presented. In section 3 mathematical operators for the root-mean-square value of a periodical signal and for the active power are introduced. In section 4 reactive power compensation is formulated as an optimization problem using two criteria, the mean-square value of the joint torque and the so-called "power factor". In section 5 a geometrical interpretation of the power factor is given. In section 6 an optimality condition valid for the two criteria is deduced. In section 7 two numerical examples are presented. One of them illustrates the fundamental limitations on reactive power compensation and the other one shows the negative effect it can have the harmonic content of the reference trajectory in energy consumption. Last Section is devoted to conclusions and perspectives.

2 Problem statement

The problem addressed in this paper is how to optimize the energy transfer between a motion actuator and a mechanical load. In robotic systems, for example, the load corresponds to the mechanical structure of the robot and the motion actuator to an electric, hydraulic, pneumatic or any other type of device suppling the joint torque necessary to produce the desired motion. The compensator system can be a torsional spring or any other elastic element capable of storing energy.

The mechanical load is supposed to be functioning as the feedback system presented in Figure 1. $q_a^d(t)$ is the desired motion for the actuated joints, Σ_l is the dynamical system representing the mechanical load, $\Sigma_c(\theta)$ is a non-dissipative passive system called mechanical compensator and Ω is a given closed loop controller

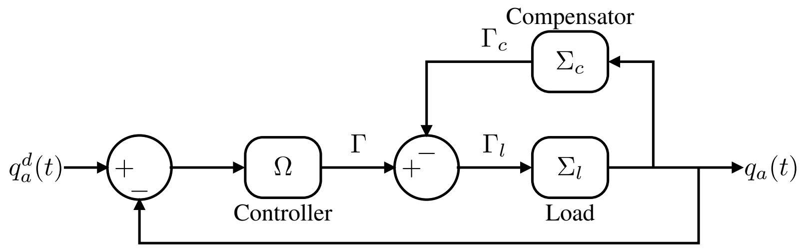

Figure 1. Closed loop mechanical system.

For the closed loop system of Figure 1 the following assumptions will be made

- $\mathbf{A1}$: The reference periodic motion $q_a^d(t)$ is a vector of smooth signals with a common period T_o.
- $\mathbf{A2}$: The controller Ω ensures the convergence of $q_a(t)$ to $q_a^d(t)$.
- $\mathbf{A3}$: The closed loop system is considered to be functioning for a long time before $t = 0$. For $t \geq 0$, $q_a(t)$ is considered to be converged to $q_a^d(t)$. In such a case, it is said that the system has reached the steady state.
- $\mathbf{A4}$: The mechanical load Σ_l is supposed to be a passive dynamical system [7]. Under the assumptions $\mathbf{A1}$ and $\mathbf{A2}$ passivity implies that the average power delivered by the actuator in a cycle is nonnegative
$$\frac{1}{T_o}\int_0^{T_o} \Gamma_l^{\mathrm{T}}(t)\,\dot{q}_a^d(t)\,\mathrm{d}t \geq 0 \tag{1}$$
- $\mathbf{A5}$: The mechanical compensator is composed by non-dissipative passive elements. Under the assumptions $\mathbf{A1}$ and $\mathbf{A2}$, it implies
$$\frac{1}{T_o}\int_0^{T_o} \Gamma_c^{\mathrm{T}}(t)\,\dot{q}_a(t)\,\mathrm{d}t = 0 \tag{2}$$

3 Mathematical notation

Given two periodical vector valued signals $x(t) \in I\!R^n$ and $y(t) \in I\!R^n$ with a common fundamental period T_o, the application of the binary operator $<\,,\,>$ to $x(t)$ and $y(t)$ gives a real quantity defined as

$$\langle x(t), y(t)\rangle \triangleq \frac{1}{T_o}\int_0^{T_o} x^{\mathrm{T}}(t)\,y(t)\,\mathrm{d}t \tag{3}$$

Using this operator, assumptions $\mathbf{A4}$ and $\mathbf{A5}$ can be written as $\left\langle \Gamma_l(t), \dot{q}_a^d(t) \right\rangle \geq 0$ and $\left\langle \Gamma_c(t), \dot{q}_a^d(t) \right\rangle = 0$. The value $\langle x(t), x(t) \rangle$ corresponds to the mean-square value of the vector signal $x(t)$

$$\langle x(t), x(t) \rangle = \frac{1}{T_o} \int_0^{T_o} x^{\mathrm{T}}(t)\, x(t)\, \mathrm{d}t, \tag{4}$$

and by consequence $\sqrt{\langle x(t), x(t) \rangle}$ is the root-mean-square (*rms*) value of $x(t) \in I\!R^n$. In the sake of simplicity, instead of $\sqrt{\langle x(t), x(t) \rangle}$, the *rms* value is denoted as follows

$$||x(t)|| \triangleq \sqrt{\langle x(t), x(t) \rangle} \tag{5}$$

4 Optimization criteria

In this section two different criteria for the minimization of steady state energy consumption are presented. Optimization will be made with respect to θ, a vector containing the parameters of the compensator system Σ. For example, if Σ is a torsional spring, θ is its stiffness.

The first criterion is given by

$$||\Gamma||^2 \triangleq \frac{1}{T_o} \int_0^{T_o} \Gamma^{\mathrm{T}}(t)\, \Gamma(t)\, \mathrm{d}t \tag{6}$$

Despite of the widespread utilization of (6) as a performance index for trajectory generation in robotic systems, its main inconvenient is the difficulty to assert when a given $||\Gamma||^2$ is small enough for a given motion. A particular value of $||\Gamma||^2$ could be considered small for certain motions but not for others. This fact does not allow a proper comparison between motions with different T_o. It would be "unfair" to compare slow and fast motions just in terms of $||\Gamma||^2$ even if they are applied to the same system. The other criterion we present in this section is known in electrical engineering as power factor [4]. The equivalent of the definition presented in [4] for mechanical systems is

$$p_f = \frac{\mathrm{P}}{\mathrm{S}} \tag{7}$$

with

$$\begin{aligned} \mathrm{P} &\triangleq \left\langle \Gamma,\, \dot{q}_a^d \right\rangle \\ \mathrm{S} &\triangleq \sum_{i=1}^{n} ||\Gamma_i|| \cdot ||\dot{q}_{a_i}^d|| \end{aligned} \tag{8}$$

$||\Gamma_i||$ and $||\dot{q}_{a_i}^d||$ being the i-th component of the vectors $\Gamma \in I\!R^n$ and $\dot{q}_a^d \in I\!R^n$. The scalar quantities P and S, respectively known as active power and apparent power [2], satisfy the Cauchy-Schwarz inequality

$$-\mathrm{S} \leq \mathrm{P} \leq \mathrm{S} \tag{9}$$

As $\Gamma = \Gamma_l + \Gamma_c$ (see Figure 1), active power can be rewritten as

$$\mathrm{P} = \left\langle \Gamma_l,\, \dot{q}_a^d \right\rangle + \left\langle \Gamma_c,\, \dot{q}_a^d \right\rangle \tag{10}$$

Under the assumption $\mathbf{A4}$ the term $\left\langle \Gamma_c,\, \dot{q}_a^d \right\rangle$ is zero and by consequence active power P does not depend on the compensator system

$$\mathrm{P} = \left\langle \Gamma_l,\, \dot{q}_a^d \right\rangle \tag{11}$$

Under the assumption $\mathbf{A5}$ the term $\left\langle \Gamma_l,\, \dot{q}_a^d \right\rangle$ is a nonnegative quantity. By consequence the inequality (9) becomes

$$0 \leq \mathrm{P} \leq \mathrm{S} \tag{12}$$

The above inequality implies that power factor is a quantity between 0 and 1. The main advantage of power-factor is that is a normalized quantity. When this quantity is close to zero, most part of the energy transfered to the load is stored and subsequently forwarded to the actuator. This phenomenon entails two main problems. The first one is that most actuators do not have energy recovery capabilities so this forwarded energy is lost by Joule effect. The second one is that the mean-square value of the torque required to produce a given motion is unnecessarily incremented. Conversely, an unitary power factor implies that for all time t power goes from the actuator to the load. These aspects will be explained in the next section.

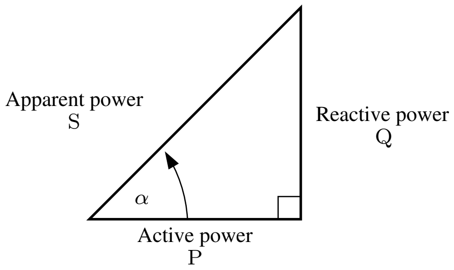

Figure 2. Power factor is the cosine of α. When reactive power is zeroed, then power factor equals to one.

5 Understanding power factor

Power factor can be understood through the right-angled triangle presented in Figure 2. The hypotenuse represents the apparent power, the horizontal cathetus the active power and the cosine of the angle between them is the power factor. The vertical cathetus is known as reactive power.

From Figure 2 it can be seen that a reduction in the reactive power leads to an improvement of the power factor. To illustrate this idea, the closed loop system presented in Figure 1 will be considered. The mechanical load Ω_l is supposed to be a linear single actuated system $\Gamma_l = J\,\ddot{q} + f_v\,\dot{q}$ (J and f_v being the inertia moment and the viscous friction coefficient). The compensator is supposed to be a torsional spring described by $\Gamma_c = k\,q$. Under the assumption $\mathbf{A3}$, the closed system can be described as

$$\Gamma = \underbrace{J\,\ddot{q}_a^d + f_v\,\dot{q}_a^d}_{\Gamma_l} + \underbrace{k\,q_a^d}_{\Gamma_c} \tag{13}$$

For this system active and apparent power are given by

$$\begin{aligned} \mathrm{P} &= < J\,\ddot{q}_a^d + f_v\,\dot{q}_a^d + k\,q_a^d,\ \dot{q}_a^d > \\ \mathrm{S} &= ||J\,\ddot{q}_a^d + f_v\,\dot{q}_a^d + k\,q_a^d|| \cdot ||\dot{q}_a^d|| \end{aligned} \tag{14}$$

Using Fourier series it can be proved that terms $< \dot{q}_a^d, q_a^d >$ and $< \dot{q}_a^d, \ddot{q}_a^d >$ are zero for any periodic signal $q_a^d(t)$. In such a case the expressions for P and S can be simplified as

$$\begin{aligned} \mathrm{P} &= f_v\,||\dot{q}_a^d||^2 \\ \mathrm{S} &= \sqrt{J^2\,||\ddot{q}_a^d||^2 + f_v^2\,||\dot{q}_a^d||^2 + k^2\,||q_a^d||^2 + 2\,J\,k < \ddot{q}_a^d, q_a^d >} \cdot ||\dot{q}_a^d|| \\ &= \sqrt{f_v^2\,||\dot{q}_a^d||^2 + ||J\,\ddot{q}_a^d + k\,q_a^d||^2} \cdot ||\dot{q}_a^d|| \\ &= \sqrt{(\,f_v\,||\dot{q}_a^d||^2\,)^2 + (\,||J\,\ddot{q}_a^d + k\,q_a^d|| \cdot ||\dot{q}_a^d||\,)^2} \\ &= \sqrt{\mathrm{P}^2 + \mathrm{Q}^2} \end{aligned} \tag{15}$$

with

$$\mathrm{Q} = ||J\,\ddot{q}_a^d + k\,q_a^d|| \cdot ||\dot{q}_a^d|| \tag{16}$$

Depending on the harmonic content of $q_a^d(t)$, a positive constant k minimizing Q can be found. If k is such that $J\,\ddot{q}_a^d + k\,q_a^d = 0$ for all t, then Q is zeroed and by consequence the power factor becomes unitary. In such a case, the compensated mechanical load is $\Gamma = f_v\,\dot{q}_a^d$, which is equivalent to a pure viscous friction element. $\Gamma = f_v\,\dot{q}_a^d$ implies that instantaneous power remains non negative for all time $t \geq 0$ and that energy flows in one direction, from the actuator to the load.

6 Optimality conditions

Firstly, we deduce a condition for the maximization of the power factor with respect to θ (a vector containing the parameters of the compensator Σ). Equation (11) shows that the active power P is independent of θ.

By consequence, the minimization of the apparent power S leads to the maximization of the quotient P/S, which is defined as the power factor. Apparent power can be written in the following way

$$\begin{aligned}
\mathrm{S} &= \sum_{i=1}^{n} ||\Gamma_i|| \cdot ||\dot{q}_{a_i}^d|| \\
&= \sum_{i=1}^{n} ||\Gamma_{l_i} + \Gamma_{c_i}|| \cdot ||\dot{q}_{a_i}^d|| \\
&= \sum_{i=1}^{n} \sqrt{||\Gamma_{l_i} + \Gamma_{c_i}||^2} \cdot ||\dot{q}_{a_i}^d|| \\
&= \sum_{i=1}^{n} \sqrt{||\Gamma_{l_i}||^2 + ||\Gamma_{c_i}||^2 + 2\,\langle \Gamma_{l_i}, \Gamma_{c_i} \rangle} \cdot ||\dot{q}_{a_i}^d||
\end{aligned} \tag{17}$$

Conversely, the apparent power for the uncompensated system ($\Gamma_c(t) \equiv 0$) is given by

$$\mathrm{S}_u = \sum_{i=1}^{n} \sqrt{||\Gamma_{l_i}||^2} \cdot ||\dot{q}_{a_i}^d|| \tag{18}$$

If we compare the expressions for S and S_u, it can be concluded that the following condition guarantees $\mathrm{S} < \mathrm{S}_u$,

$$||\Gamma_{c_i}||^2 + 2\,\langle \Gamma_{l_i}, \Gamma_{c_i} \rangle < 0 \qquad i \in \{1, \ldots, n\}, \tag{19}$$

If the above inequalities are satisfied for all i, then apparent power of each actuator is decreased and so the total apparent power S. Condition (19), however, is sufficient but not necessary. The total apparent power S could be decreased even if the above inequalities are satisfied for some (but not all) values of i. In the case of a single-actuated system ($n = 1$) condition (19) becomes both necessary and sufficient.

Now we deduce a necessary and sufficient condition for the minimization of the mean-square joint torque. The joint torque supplied by the actuator is decomposed as the sum of Γ_l and Γ_c (see Figure 1)

$$\begin{aligned}
||\Gamma||^2 &= \frac{1}{T_o} \int_0^{T_o} \left[\Gamma_l(t) + \Gamma_c(t)\right]^{\mathrm{T}} \left[\Gamma_l(t) + \Gamma_c(t)\right] \,\mathrm{d}t \\
&= \frac{1}{T_o} \int_0^{T_o} \Gamma_l^{\mathrm{T}}(t)\,\Gamma_l(t) + \Gamma_c^{\mathrm{T}}(t)\,\Gamma_c(t) + 2\Gamma_l^{\mathrm{T}}(t)\,\Gamma_c(t)\mathrm{d}t
\end{aligned} \tag{20}$$

Using the operators $||.||$ and $<,>$ the above equation can be rewritten as

$$||\Gamma||^2 = ||\Gamma_l||^2 + ||\Gamma_c||^2 + 2\,\langle \Gamma_l,\, \Gamma_c \rangle \tag{21}$$

If the following inequality is satisfied

$$||\Gamma_c||^2 + 2\,\langle \Gamma_l,\, \Gamma_c \rangle < 0, \tag{22}$$

or equivalently

$$\sum_{i=1}^{n} ||\Gamma_{c_i}||^2 + 2\,\langle \Gamma_{l_i},\, \Gamma_{c_i} \rangle < 0, \tag{23}$$

then $||\Gamma||^2 < ||\Gamma_l||^2$. In such a case, the mechanical compensator leads to a less energy consumption in the sense of the criterion (6). Criteria (19) and (23) are equivalent only for single-actuated systems.

7 Numerical simulations

In this section two numerical examples are presented. The first one illustrates the fundamental limitations on reactive power compensation and the other one shows the negative effect that can have the harmonic content of the reference trajectory in energy consumption.

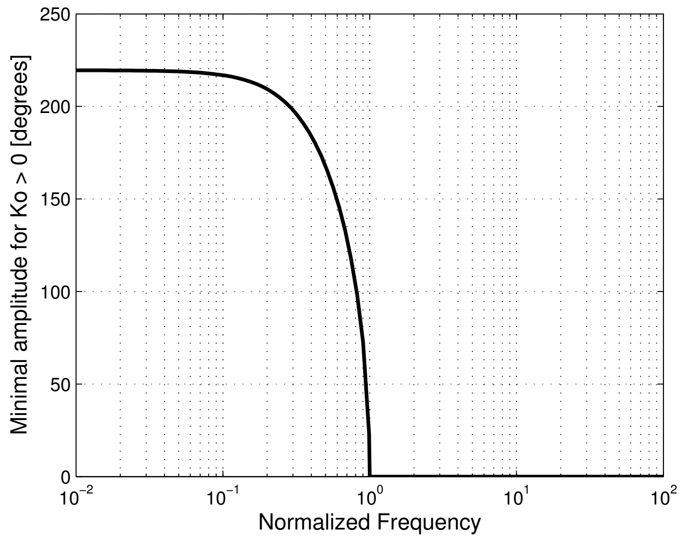

Figure 3. Amplitude-Frequency optimality condition. Minimal motion amplitude required to obtain a positive optimal spring constant. Frequency is normalized with respect to the natural frequency of the pendulum.

Example 1 Consider the closed-loop mechanical system presented in Figure 1 and suppose that Σ_l is a single pendulum system, Σ_c is a series torsional spring and Ω a control law satisfying the assumption **A3**. The parameters of the pendulum in the international units (MKS) are $J = 0.981$ (inertia), $m = 0.1$ (mass), $l = 1$ (length), $f_v = 0.1$ (viscous friction) and $g = 9.81$ is the gravity force. The objective is to maximize the power factor by optimizing the stiffness of the spring. The desired motion is supposed to be given by

$$q_a^d(t) = A \sin(\omega_o t) \tag{24}$$

The steady state closed loop dynamics can be described as follows

$$\begin{aligned} \Gamma_l &= J \ddot{q}_a^d + m g l \sin\left(q_a^d\right) + f_v \dot{q}_a^d \\ \Gamma_c &= k q_a^d \end{aligned} \tag{25}$$

For small motion amplitude $\sin\left(q_a^d\right)$ can be approximated by q_a^d. Under this assumption it can be shown that the optimality condition (19) leads to

$$k < 2\left(J \omega_o^2 - m g l\right) \tag{26}$$

The above inequality implies that power factor can be improved using a torsional spring only when frequency motion is greater than the natural frequency of the pendulum $\omega_n = \sqrt{m g l / J}$, otherwise problem is infeasible because a negative k is required. If the reference motion does not allow to approximate $\sin\left(q_a^d\right)$ by q_a^d the optimality condition (19) gives an upper limit for k depending on A and ω_o. Unfortunately, an expression in a closed form like (26) cannot be obtained in that case. The graph presented in Figure 3 shows the minimal value of A required to obtain a non-negative upper limit for k for a given frequency ω_o. If the pair (A, ω_o) is below the curve, the optimality condition (19) cannot be satisfied for any positive value of k. If (A, ω_o) is above the curve, there exist a set of positive values of k leading to an improvement of the power factor. It is interesting to note that for frequencies higher than the natural frequency, optimization is possible for all amplitudes.

As seen in Section 4 if the compensated mechanical load is seen by the actuator as pure viscous friction element, then the power factor becomes unitary. Using the equation (25) it can be concluded that $\Gamma = f_v \dot{q}_a^d$ can be obtained, if and only if, there exist a constant value of k satisfying

$$J \ddot{q}_a^d + m g l \sin\left(q_a^d\right) + k q_a^d = 0, \quad \forall t \geq 0 \tag{27}$$

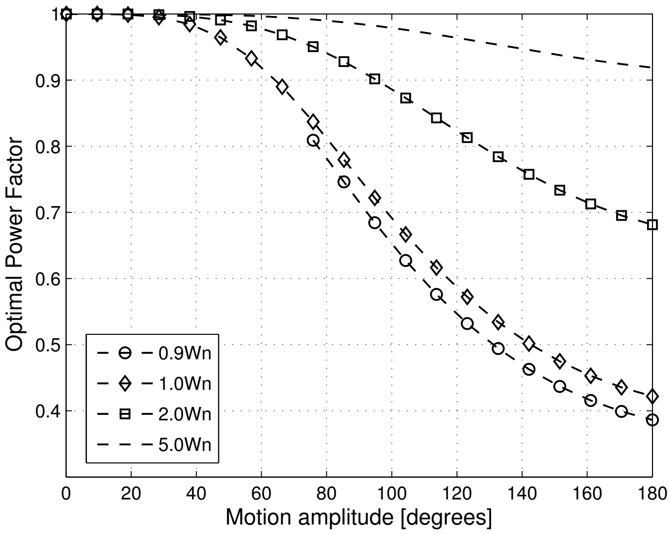

Figure 4. Maximal power factor in a single pendulum system when the reference motion is a sinusoidal signal of given amplitude.

The above equation can be satisfied for a constant k only if the amplitude of the desired motion is small. In such a case, k is given by $k = J\omega_o^2 - m\,g\,l$. If $\sin\left(q_a^d\right)$ cannot be approximated by q_a^d then an unitary optimal power factor cannot be obtained. Figure 4 shows the maximal power factor as a function of the amplitude and frequency. From this figure it can be observed that for a fixed frequency if the amplitude of the desired motion is augmented then the maximal power factor that can be obtained decreases. Conversely, for a motion of fixed amplitude, greater is the frequency, greater is the maximal power factor.

Example 2 Reference motion in robotic systems is often indicated by using only initial and final conditions on joint positions and velocities. This implies that an infinite number of time functions satisfying such conditions can be generated. For example, for the system of the previous example, the two periodic motions presented in Figure 5 satisfy the conditions $q(0) = -20^o$, $q(T/2) = 20^o$, $\dot{q}(0) = 0$, $\dot{q}(T/2) = 0$. Both reference motions have a fundamental frequency equal to $\omega_o = 10$ rad/seg which is ten times the natural frequency of the pendulum ($\omega_n = \sqrt{m\,g\,l/J}$). One of the motions is defined as a single frequency sinusoidal signal and the other one is obtained by concatenating two polynomials of third order.

- Sinusoidal motion:

$$q_a^d(t) = A\,\sin\left(\omega_o\,t\right) \tag{28}$$

- Polynomial motion:

$$q_a^d(t) = \begin{cases} a_3t^3 + a_2t^2 + a_1t + a_0, & 0 \le t < \frac{T_o}{2} \\ b_3t^3 + b_2t^2 + b_1t + b_0, & \frac{T_o}{2} \le t < T_o \end{cases} \tag{29}$$

As motions are almost identical it could be expected that the joint torque curves for the polynomial and the sinusoidal references be also quite similar. However, this is not always true. It depends on the transfer function $\Gamma(s)/q_a^d(s)$. If $\Gamma(s)/q_a^d(s)$ has a strong gain at high frequencies, some of the high-order harmonic components of the polynomial motion could have a more important gain that the fundamental component. In such a case, the torques could be quite different even if motions are very similar. If the high frequency gain of $\Gamma(s)/q_a^d(s)$ is limited, the convergence of $q_a(t)$ towards $q_a^d(t)$ can be seriously affected. In summary, there is trade-off between tracking and energy consumption. To illustrate this point a linearized version of

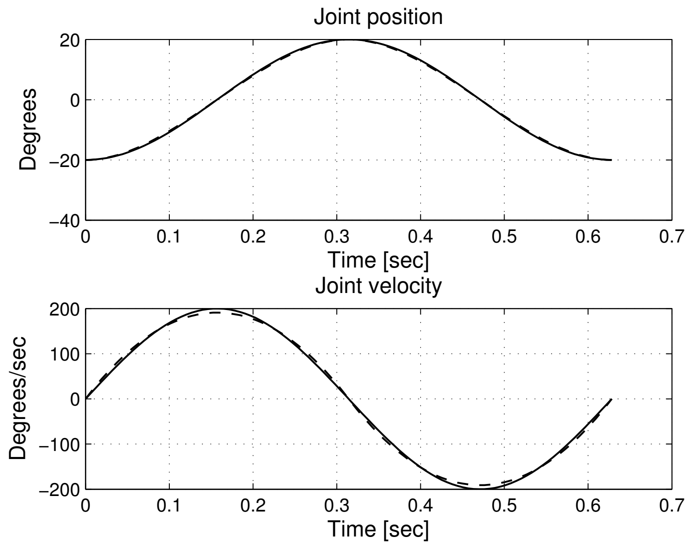

Figure 5. Reference motions for the example 2. One of them is defined as a single frequency sinusoidal signal (solid line) and the second one by concatenating two polynomials of third order (dashed line).

the pendulum system will be considered

$$\Gamma = J\,\ddot{q}_a^d + f_v\,\dot{q}_a^d + (m\,g\,l + k)\;q_a^d \tag{30}$$

By taking the Laplace transform of the last equation, the following transfer function is obtained

$$\frac{\Gamma(s)}{q_a^d(s)} = J\,s^2 + f_v\,s + (m\,g\,l + k) \tag{31}$$

This transfer function is unrealistic because the resulting steady-state gain increases indefinitely as frequency increases. In practice, closed-loop steady-state gain is limited by the actuator dynamics. To obtain a more convincing transfer function the following considerations will be made

- Actuator is supposed to be modeled as a low-pass filter with a cutoff frequency of $10\omega_o$ (ω_o being the fundamental frequency of the reference motion)

$$G_a(s) = \frac{1}{(0.01s+1)^2} \tag{32}$$

- Controller Ω (see Figure 1) is represented by a classical lead compensator [1]

$$G_c(s) = 5\,\frac{1.01s+1}{0.01s+1} \tag{33}$$

With the above considerations the steady-state gain of the transfer function $\Gamma(s)/q_a^d(s)$ decreases as frequency increases when $\omega > 10\omega_o$. (Figure (6)). For frequencies between ω_o and $10\omega_o$ the gain increases as frequency increases. From this figure it can be seen that harmonic components with frequencies between $2\omega_o$ and $10\omega_o$ have a gain more than ten times larger than the gain of the fundamental harmonic ω_o. As consequence, even if the two motions presented in Figure 5 are very similar, the corresponding instantaneous power curves are very different (see Figure 7). From this Figure it can be seen that for the sinusoidal motion instantaneous power remains non-negative, and for the polynomial motion power oscillations are important. For the sinusoidal motion power factor is unitary and $||\Gamma||^2 = 0.0369$, for the polynomial motion power factor is 0.4569 and $||\Gamma||^2 = 0.1769$.

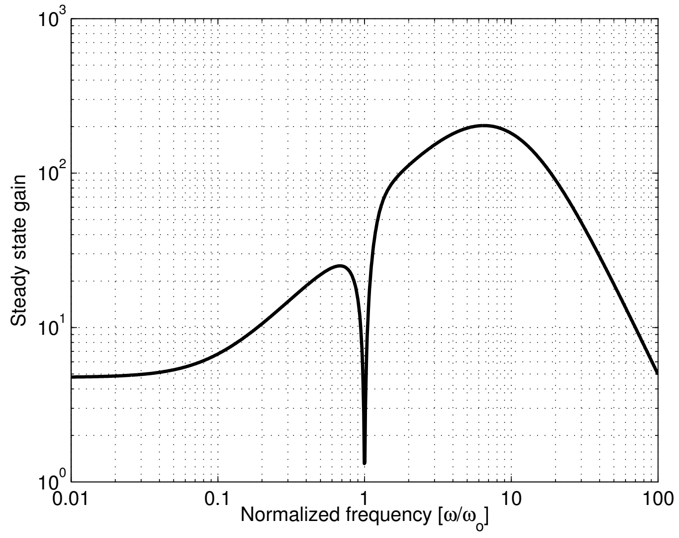

Figure 6. Steady state gain of the closed loop transfer function $|\Gamma(j\omega)/q_a^d(j\omega)|$ when actuator dynamics is considered.

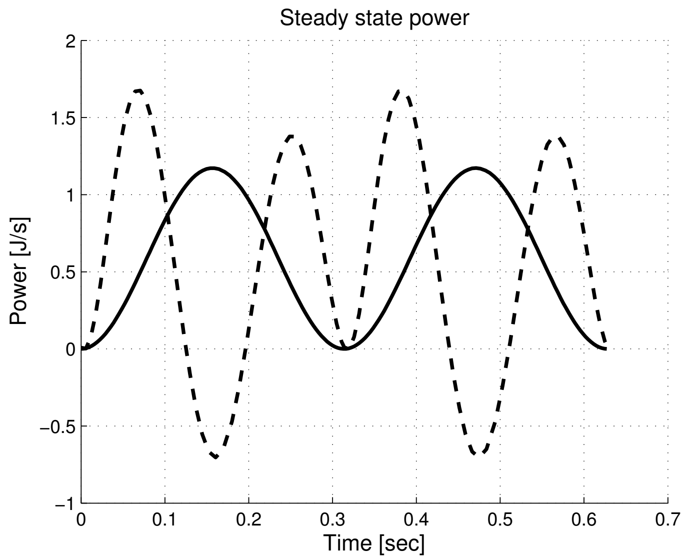

Figure 7. Instantaneous power for the sinusoidal motion (solid line) and for the polynomial motion (dashed line).

8 Conclusions and perspectives

The problem of the steady state reactive power compensation in closed loop mechanical system subject to periodic motion has been presented. Compensation is done by using non-dissipative passive elements like torsional springs. Parameter selection for these elements is formulated as an optimization problem. Two different performance index are presented. One of them is the classical mean-square value of the torque and the other one, inspired from the electrical networks theory, is known as power factor. This latter is a normalized quantity between 0 and 1 which depends on the torque and on the desired motion. It has the advantage to allow the comparison between systems with different motions.

Based on the idea of optimization of power factor by the use of capacitors in electrical system, the use of springs is proposed to optimize the "energy" consumption in mechanical system. Since the stiffness is a positive coefficient the efficiency of this approach depends on the desired trajectory. In the example 1, the positiveness of the spring stiffness depends on the amplitude and the frequency of the motion. An unitary power factor can be obtained when the compensated mechanical load is seen by the actuator as a linear viscous friction element of the form $\Gamma = f_v\,\dot{q}_a^d$. In such a case instantaneous power remains on-negative for all time $t \geq 0$ and energy flows in one direction, from the actuator to the load.

Another aspect studied in the paper is the effect of the harmonic content of the reference motion in the power factor of the system. In the Example 2, it is shown that very similar reference motions can produce very different power factors. In the cited example, this phenomenon is explained by the amplification of the high order harmonic components of the reference motion.

The feedback system presented in Figure 1, allows to include springs only in active joints. Our main perspective is then to develop a conceptual framework allowing to study more general interconnections between mechanical systems and passive compensators. It would be interest, for example, to understand the effect of passive arms in the energy consumed by a bipedal robot.

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